EG&G PRINCETON APPLIED RESEARCH

Scientific Instrument Group

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Technical Note 151A

The Use of a Lock-In Amplifier for the Detection and Measurement of Light Signals

Introduction

When one is interested in precisely recovering a signal from noise, the Lock-In Amplifier method is often very useful. In this method, one chops or interrupts the signal at a frequency appreciably above the information carrying frequencies of the signal, amplifies at this chopping frequency, and then synchronously demodulates this amplified output in order to recover the original signal information. The method is generally applicable to the situation where the desired signal information is slowly changing with time, and achieves its power from the fact that amplification at the chopping frequency can be done in a much more noise-free way than at the signal information frequencies where practical amplifiers are contaminated with 1/f noise. It is often possible to choose to conduct an experiment slowly in cases where noise is a problem; this places the desired signal information in a very narrow band, and allows narrow band filtering to be used to reject the noise. The Lock-In Amplifier achieves its narrow band filtering at DC where narrow band filters are easy to construct (*viz*, RC networks).

The purpose of this note is to discuss the considerations involved in the application of the Lock-In Amplifier technique to the processing of the signals derived from the detection of light with a photomultiplier.

Nomenclature and Definitions

A photomultiplier consists of a light-sensitive surface, called the photocathode, that for each incident photon has a certain probability of emitting an electron. This probability is known as the quantum efficiency, depends upon the type of surface and the wavelength of the incident light, and in the most favorable cases may reach a value as high as 0.25. The photoelectrons are accelerated by an internal electrostatic field and caused to strike a specially treated electrode called a dynode, where several electrons are produced by the process of secondary electron emission. These electrons in turn are accelerated to a second dynode where the multiplication process is repeated. An appreciable number of repetitions of this process eventually yields a burst of between 10^5 to 10^7 electrons that are collected at the final electrode, the anode, for each initial photoelectron. The following symbols will be used in the analysis and discussion:

Symbol	Meaning	Dimensions
f_s	Signal photoelectron rate	sec^{-1}
f_d	Dark photoelectron rate	sec^{-1}
z	Excess noise factor	dimensionless
G	Total photomultiplier gain	dimensionless
δ	Secondary emission ratio in photomultiplier	dimensionless
i_s	Signal current at photomultiplier anode	amperes
e	Charge on electron	1.6×10^{19} coulombs
B	Bandwidth	Hz
R_e	Photomultiplier anode load resistance	ohms
R_i	Preamplifier input resistance	ohms
R	Parallel combination of R_e and R_i	ohms
k	Boltzmann constant	1.38×10^{-23} joules/°K
T	Temperature	degrees Kelvin

Noise in Photomultipliers

The two sources of noise in a photomultiplier that are usually dominant in the measurement of weak light fluxes are the fluctuation noises in the dark current and in the signal current. A single photoelectron emitted from the photocathode yields on the average a packet of charge Ge at the anode. The signal current at the anode is given by $i_s = f_sGe$ and the noise current at the anode is given by

$$i_n^2 = 2(f_s + f_d)\, G^2e^2B \;\; \frac{\delta}{\delta-1}$$

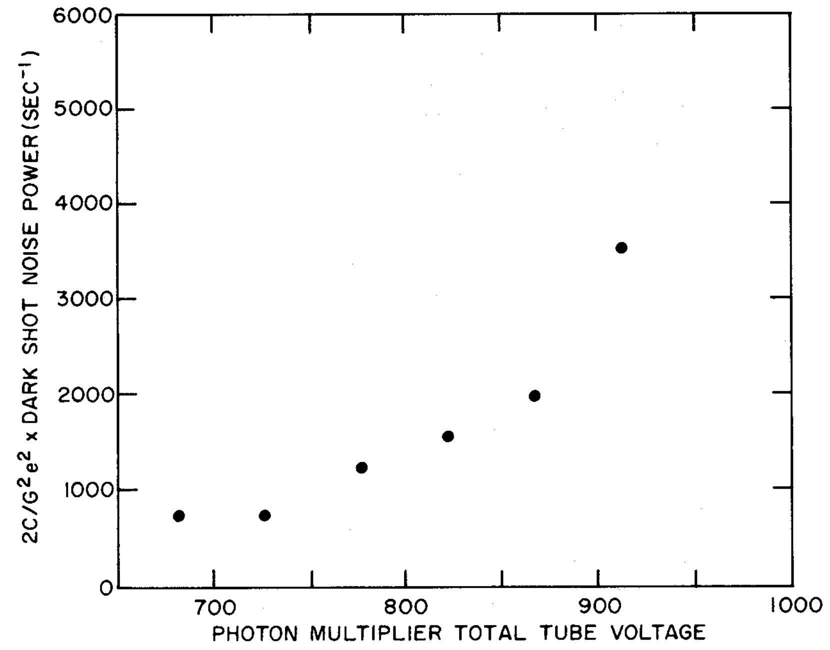

FIGURE 1. Effect of photomultiplier voltage on dark current noise power.

The final factor $\frac{\delta}{\delta-1}$ comes from the statistical fluctuations in the secondary emission process; the main factor is just the classical shot noise formula with the amplified charge Ge replacing the electronic charge.

Effects such as variation of multiplier gain for different electron trajectories, stages, and dynode surface patches cause the actual noise to exceed that given by the above formula; these effects may be lumped together by replacing the factor $\frac{\delta}{\delta-1}$ by an excess noise factor z which is greater than 1, and may be as high as 4 for a poorer tube.

This noise formula treats the dark current noise as though it were wholly due to electrons emitted from the photocathode. However, emission from later stages will also contribute to the noise. If the dark emission from intermediate dynodes is of the same order as that from the photocathode, the lesser gain experienced by the emission from these later dynodes makes their noise contributions smaller. A slight upward adjustment of the dark electron rate f_d enables the formula to include these effects.

More serious is the effect of field emission from sharp points or whiskers in the dynode structure. This type of emission increases rapidly with voltage, giving a large noise-producing current. The noise pulses at the anode can exist in great profusion, but being small may be missed in a cursory examination of the output signal. A careful examination of the dark current noise power normalized by dividing by the square of the photomultiplier gain will often show a sharp increase at relatively modest total photomultiplier voltages. It is usually desirable to operate at voltages well below this break. Figure 1 shows this effect for an EMI 9660 B photomultiplier, where it is apparent that above 750 volts the noise begins to rise significantly. Up to the highest voltages tried, 1100 volts, the tube gave no other indication of distress. Moral: Keep the voltage down!

Assuming that reasonable care has been taken to reduce or eliminate other noise sources, such as photomultiplier socket leakage, electric or magnetic fields about the photomultiplier that perturb its normal operation, light leaks, and electrical interference pickup, and to arrange that the amplifier does not add excessive noise so the intrinsic photomultiplier noise is dominant, the signal-to-noise ratio at the output of a Lock-In Amplifier may be calculated. For the case where the modulation of the light signal is a square wave, and where the amplifier passes this wave to the mixer board undistorted:

$$S/N = \frac{f_s}{2\sqrt{(f_s + 2f_d)\,Bz}}$$

With a tuned amplifier selecting the fundamental component of the square wave signal

$$S/N = \sqrt{\frac{2}{\pi}}\;\frac{f_s}{(f_s + 2f_d)\,Bz}$$

The expressions are for the ratio of signal amplitude to noise amplitude. The first method is 10% better in signal-to-noise.

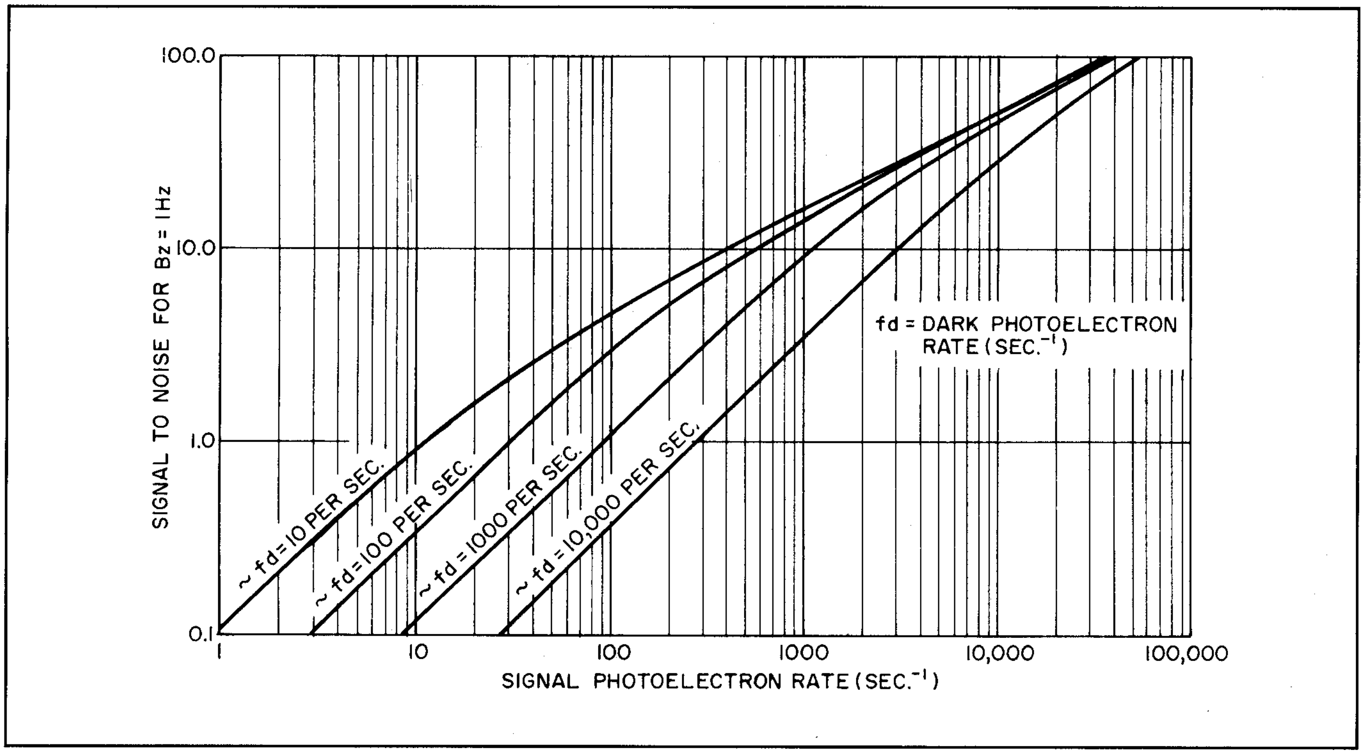

FIGURE 2. Signal-to-noise as a function of signal photoelectron rate with zB = 1 Hz.

The march of the signal-to-noise with varying f_s and f_d may be illustrated more clearly by putting

$$\alpha = \frac{f_d}{f_s}$$

Then we may write

$$S/N = \frac{\sqrt{f_s}}{2\sqrt{(1 + 2\alpha)}} \; \frac{1}{\sqrt{zB}}$$

or

$$S/N = \frac{f_s/\sqrt{f_d}}{2\sqrt{\frac{1}{\alpha} + 2}} \; \frac{1}{\sqrt{zB}}$$

The first form is useful if $f_s >> f_d$; then α is small, and the signal-to-noise goes as the square root of the signal. If $f_d >> f_s$ then the second form is more convenient, and shows the signal-to-noise is proportional to the signal. Figure 2 is a plot of S/N as a function of f_s with zB = 1 Hz for various values of the dark electron rate. The curves all have the same shape. An increase of dark photoelectron rate by a factor of 10 corresponds to one decade translation to the right, and one half decade translation upwards of the original curve. Interpolation is thus relatively easy, but direct substitution in the formula is more convenient.

Coupling the Photomultiplier to the Amplifier

In general, one is able to arrange that the noise added by the load resistor and the amplifier is small compared to the intrinsic photomultiplier noise. A noise equivalent circuit is shown in Figure 3. The noise current generator i_1 is that associated with the photomultiplier, i_2 is the Johnson noise current in the resistor R which is the parallel combination of the photomultiplier anode load resistor and the input resistance of the amplifier, i_3 is the noise current generator, and e_1 is the noise voltage generator associated with the active amplifying element in the amplifier.

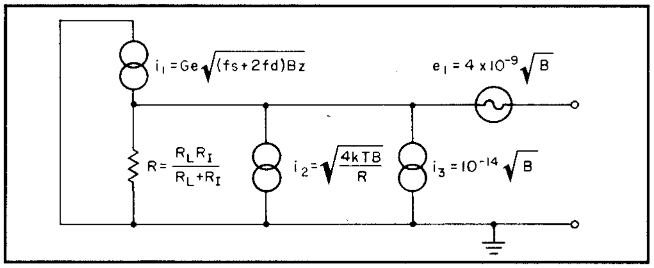

FIGURE 3. Noise equivalent circuit.

A comparison of the various noise currents at unit bandwidth (1 Hz) is instructive. For:

$$G = 200{,}000$$

$$f_s + 2f_d = 1000/\text{sec.}$$

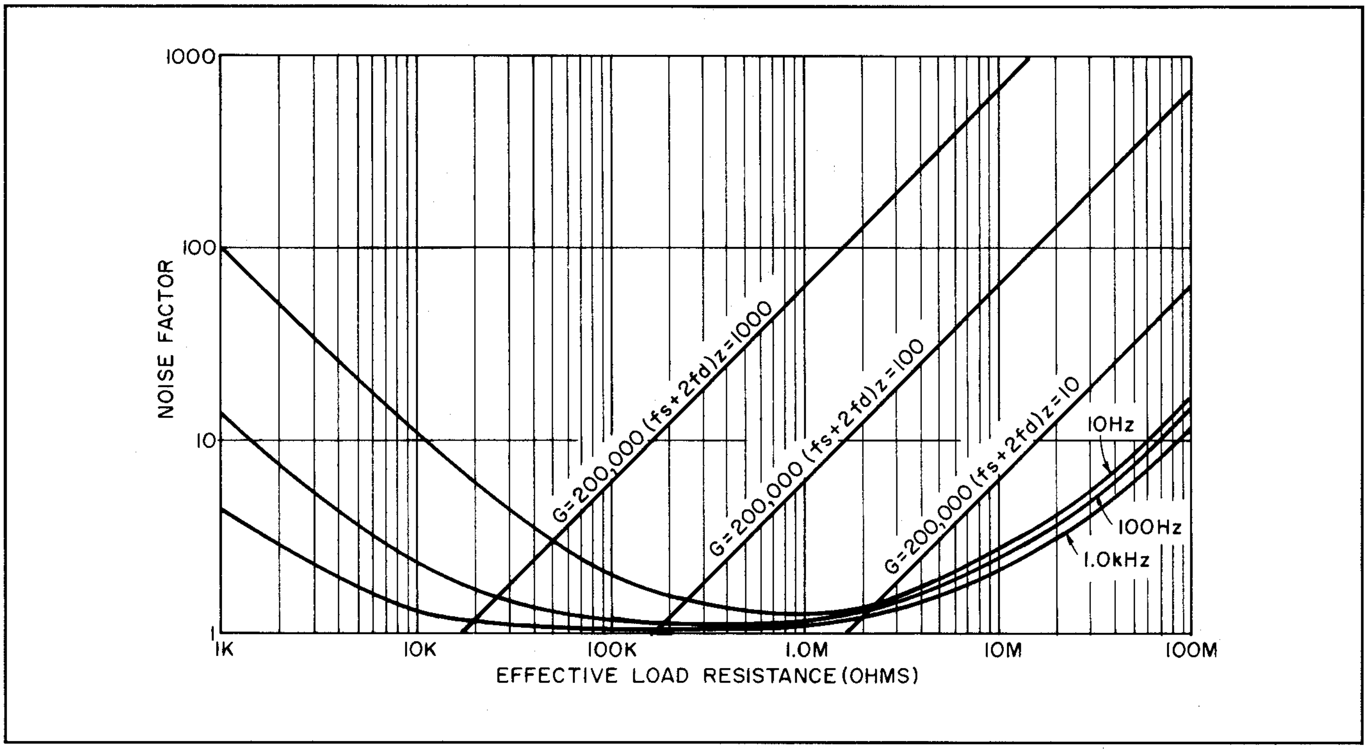

FIGURE 4. Noise factor as a function of source resistance.

T = 300°K
i_1 = 10^{-12}amperes/root cycle
i_2 = $1.29 \times 10^{-10}/\sqrt{R}$ amperes/root cycle
i_3 = 10^{-14} amperes/root cycle
e_1 = 4×10^{-9} volts/root cycle

It is clear that i_1, the noise current from the photomultiplier, exceeds greatly the internal noise current generator of the amplifier, and appreciably the Johnson noise of an equivalent load resistance greater than 100K. If the tube were refrigerated and unusually quiet, and operating with a very weak input light signal, $f_s + 2f_d$ might be as low as 10/sec, dropping i_1 to 10^{-13}amperes/root cycle, still exceeding i_3 by a factor of ten, and the noise current i_2 if R is 10 megohms or greater.

The contribution of the noise voltage source e_1 relative to the photomultiplier effective output voltage i_1R clearly diminishes as R increases. Sometimes the photomultiplier anode load is erroneously chosen to be the source resistance for which the amplifier shows the best noise figure. If one has a source of signal power whose impedance level is adjustable in a lossless way, either by transducer design or impedance transformation, then it is correct to choose this impedance to be that for which the amplifier has the best noise figure. The photomultiplier, however, is a signal current source, and the higher the load resistance placed upon it, the more the photomultiplier internal noise and the signal, dominate the other noises.

The noise factor for the HR-8 Type A Preamplifier as a function of source resistance is plotted in Figure 4. This factor is the ratio of the total noise output power of the amplifier to the amplified Johnson noise power of the source resistance. The diagonal straight lines of 45° slope are plots of the factor by which the photomultiplier noise exceeds the Johnson noise in the equivalent anode load resistance R for a photomultiplier gain of 200,000 and for different dark and signal photoelectron rates. The highest diagonal line $(f_s + 2f_d)\, z = 1000$ is perhaps typical of a good tube operated at room temperature, and the lowest diagonal line typical of dry ice temperature operation. If the photomultiplier is operated at a higher (lower) gain, the lines should be moved upwards (downwards) by a factor which is the square of ratio of the actual gain to the gain of 200,000 for which the lines were determined. The vertical distance between the photomultiplier line and the preamplifier line gives the factor by which the photomultiplier noise exceeds the amplifier noise. If the extra noise added by the preamplifier is less than the intrinsic photomultiplier noise by a factor of 1/10 in power, or $1/\sqrt{10}$ in amplitude, then its effects are usually negligible. It is clear that high load impedances should be chosen, and the quieter the photomultiplier and the weaker the light signal, the larger this impedance should be. For most cases, an impedance between 1 and 5 megohms is satisfactory.

The Effects of Parasitic Capacity

In practice, the anode circuit of the photomultiplier is loaded by the tube output capacity of about 10 pf, the amplifier input capacity of 20 pf, and the cable capacity, for which 100 pf per meter might be typical.

Figure 4 may still be used for analyzing the noise contribution of the amplifier when the capacity loading is significant if the chart is entered with the input circuit impedance instead of the resistance. Excessive capacity has two undesirable effects: it limits the impedance that can be presented to the photomultiplier at the signal chopping frequency and causes a phase shift between the light signal and the electrical output. Both to keep the input capacity down and to reduce electrical interference pickups, the preamplifier should be placed as close as possible to the photomultiplier.

Fighting Pickup

The electric power service to the laboratory places significant amounts of energy at 60 Hz and its harmonics in the volume and flowing over the metallic surfaces of a laboratory. Some care must be taken to ensure that the coupling of the input circuits to the power line be kept small. The signal power of interest might be 10^{-19} watts, and the utility circuits are dealing with kilowatts, so the power coupling coefficient between the utility and the signal circuit needs to be less than 10^{-22}, a very small number. When dealing with pickup problems, one takes a different point of view than when one analyzes the circuits. The pickup fighter's point of view is:

1. No ground system is an equipotential.
2. All circuits have a residual area to convert varying magnetic flux into circuit voltages.
3. All electrostatic shields have holes through which electric fields leak.
4. All shields on coaxes allow energy leakage into the center conductor.
5. All other pieces of equipment, such as motors, transformers, and fluorescent lamps, were devilishly designed to spray as much energy around as possible.

Several precautions are relatively easy to take, and give considerable protection. If the preamplifier has a differential input, then run a cable that is terminated with the same impedance as is used for the anode load next to the signal cable and connect it to the other input. The photomultiplier, and its anode load resistance, the pickup matching resistance, and the signal leads should be well shielded. This arrangement protects against electrostatic pickup. It is sometimes desirable to place the anode load resistor at the preamplifier. The high internal impedance of the photomultiplier effectively opens a potential ground loop.

Magnetic pickup most commonly makes itself felt by way of ground loops. If the various ground interconnections of the elements of the system form a closed loop, quite large currents can be induced by alternating magnetic flux threading the loop. The IR drop of such a current along the shield of the signal lead will directly add to the amplifier input. Such loops should be avoided where possible.

Most laboratory utility systems are of the three-wire type, two carrying the power and the third wire grounded, usually at the utility transformer. Different laboratory circuits are often fed from different phases of the utility transformer, and the separate ground leads can be at significantly different potentials. The various instruments that are connected together to form the system are grounded each to the utility system ground by way of the third wire. If some parts of the system are plugged into one utility circuit and the others are plugged into another, the difference in potentials of the two utility grounds can drive currents of the order of several amperes through the good interconnections that are the inter-instrument signal return paths, and cause considerable interference. Special attention has been paid in the design of these instruments in order to minimize the effects of induced currents in the ground system; however, efforts spent in reducing the area of ground loops is well worthwhile.

Electric motors and undershielded or overloaded transformers are sources of significant magnetic interferences, and fluorescent lights are sources of significant electrostatic as well as magnetic interference. Universal motors such as are used to power electric hand drills create broad band noise due to commutation at the brushes which can be spread over a relatively wide frequency band. Their use in the vicinity of an important experiment should be avoided.

It is, of course, possible to use the narrow band characteristics of the Lock-In Amplifier to reject interference contributions at the power line frequency. It is generally not difficult, and is certainly a more sanitary way of doing things, to reduce the line frequency contamination to a level where its power is no greater than the irreducible system noise in a 100 Hz bandwidth. The presence of large line frequency interference squanders system dynamic range, and limits the experimenter's ability to open up the system bandwidth for quick exploration of his signals.

Measurement of Noise Performance

If a light detection system has been properly designed, and is working as it should, the signal-to-noise will be given by the relations given earlier in the discussion of noise in photomultipliers. In order to test whether the theoretical signal-to-noise is being achieved, one needs measurements of the signal photoelectron rate, f_s, and the dark photoelectron rate, f_d.

One method that is fairly convenient is to choose a weak light intensity that gives a photoelectron rate 5 to 10 times background, and to count pulses. If a suitable discriminator and scaler is not available, an oscilloscope can be randomly single swept at a writing rate that gives an average of 3-4 pulses/sweep, and the total sweep time and number of pulses for perhaps 100 sweeps accumulated. Under the same conditions, the noise power in the photomultiplier anode load is inferred from measurements of the noise power at the output of an amplifier with convenient gain and bandwidth. The bandwidth should be chosen small enough that the pulses show some rounding and overlap; this reduces the dynamic range requirements of the rms voltmeter. This calibrates the noise power meter in counts/sec. The noise power meter now can be used for measuring the photoelectron rate over a wide range, from perhaps 100/sec to the limit of linear operation of the photomultiplier.

The noise amplitude at the output of the Lock-In Amplifier may be determined by taking a sequence of readings of the output spaced apart in time by at least 2 RC, and computing the variance by standard statistical techniques. To compare the observed noise with the theoretical values, one needs the equivalent noise bandwidth of the Lock-In Amplifier averaging circuit. For the 6 dB/octave slope it is 1/4RC and for the 12 dB/octave slope it is 1/8RC.

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